

# The Principles of Calculus

## I, II, and III

Course Outline

Rough Draft - 09-29-2021

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# Course Outline – Common Features

## Common Features

Lecture: three hours per week.

Discussion sections: two hours per week. One is a collaborative learning workshop. One is a directed question and answer period.

Homework: has two parts. The first part consists of self-check problems delivered electronically. The second part consists of careful write ups of the worksheets that have previously undergone guided peer review.

Exams: One midterm and one final.

Video: Video content supplements discussion section content and should be watched prior to discussion section.

# Course Outline

## The Principles of Calculus I

Note: This course has already been constructed and is being currently implemented.

- I. Decomposition
- II. Transformation
- III. Rigidity
- IV. Symmetry

### Short Description:

The first course in the three course sequence, The Principles of Calculus I is a course on the application of transformation and symmetry to the study of elementary functions. Decomposition, Transformation, Rigidity, and Symmetry are the four themes of the course. Topics include: functions and their graphical representation; operations on and with functions; set builder notation; linear systems; the group of isometries of the plane; scalings of the plane; physical units; trigonometry and its applications; inverse functions; inversion of the  $y$ -axis; polynomial and rational functions; rigidity of rational functions; rigidity and intersections of lines and planes; graphical representations of trigonometric functions and sinusoidal functions; trigonometric equations; exponential functions; logarithms; tangency in an algebraic setting.

# Course Outline

## The Principles of Calculus II

- V. Finite Approximation
- VI. Local Linear Approximation of Functions

### Short Description:

The Principles of Calculus II applies the principle of finite approximation to the study of the local linear approximation of functions. Topics include: finite approximation of planar area, sequences and their limits, analysis of error; continuous limits; continuity; asymptotic behavior; approximating rate of change; the derivative; Newton’s Method; approximation by the tangent line; derivatives of elementary functions; implicit differentiation; related rates; the geometry of particle motion; the mean value theorem; extremal points; the antiderivative; simple first order differential equations.

# Course Outline

## The Principles of Calculus III

- VII. Local Higher Order Approximation
- VIII. Integration

### Short Description:

The Principles of Calculus III uses the higher order approximations of functions as well as the principles of decomposition and integration to study global properties of functions from their local behavior. Topics include: series and their convergence; Taylor’s theorem and Taylor series; elementary differential equations of higher degree; the geometry of particle motion in several spatial variables; the fundamental theorems of calculus I and II and their consequences; integration techniques; area, volume, arc length, and surface area integrals in a single real variable; work integrals and planar area.

# Principles of Calculus I

Note: This course has already been constructed and is being currently implemented as the current Math 5.

## • I. Decomposition

- I.1. The Algebra of Sets
  - I.1.1. Setting the Stage
  - I.1.2. The Language of Set Theory
  - I.1.3. Unions and Intersections
- I.2. Intervals and Linear Inequalities
  - I.2.1. Unions and Intersections of Intervals
  - I.2.2. Multiple Linear Inequalities
- I.3. Functions and their Basic Properties
  - I.3.1. Cartesian Products and Relations
  - I.3.2. Basic Properties of Functions
  - I.3.3. Comparing Functions
- I.4. Functions Given by Simple Formulas
  - I.4.1. Formulas for Functions
  - I.4.2. Lines
  - I.4.3. An Elementary Library
- I.5. Manipulating Functions
  - I.5.1. Restriction to Subdomains
  - I.5.2. The Algebra of Functions
  - I.5.3. Decomposing Functions
  - I.5.4. Computing the Range of a Function
- I.6. Piecewise Functions
  - I.6.1. Decomposing Domains
  - I.6.2. Compound Piecewise Defined Functions
  - I.6.3. Inequalities Involving Piecewise Defined Functions
- I.7. Functions on Subsets of the Plane
  - I.7.1. Functions on the Plane
  - I.7.2. Level Sets
  - I.7.3. Single Variable Graphs from Multivariate Functions
- I.8. Linear Systems and Feasible Sets
  - I.8.1. Systems of Linear Equations

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I.8.2. Systems of Linear Inequalities

I.8.3. Expressing Feasible Sets in Set Builder Notation

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• II. Transformation

- II.1. Vectors and Translation
    - II.1.1. Abstract Translations of the Plane
    - II.1.2. Vectors and the Method of Coordinates on a Plane
    - II.1.3. Translating Sets and Graphs
  - II.2. Scaling Vectors and Subsets of the Plane
    - II.2.1. Scaling Vectors
    - II.2.2. Circles and the Polar Form of a Vector
    - II.2.3. Scaling Subsets of the Plane
  - II.3. Scaling Quantities
    - II.3.1. Units
    - II.3.2. Linear Scaling
    - II.3.3. Simple Nonlinear Scaling
    - II.3.4. General Nonlinear Scaling
  - II.4. Movement along Lines
    - II.4.1. Absolute and Relative Movement
    - II.4.2. Parameterized Lines
  - II.5. Orthogonality and Reflection
    - II.5.1. Orthogonality of Vectors and Lines
    - II.5.2. Distance from Points to Lines
    - II.5.3. Reflecting Sets across Arbitrary Lines
  - II.6. Inverse Functions
    - II.6.1. Reflection and Inverse Functions
    - II.6.2. Restricting Domain to Guarantee Invertibility
  - II.7. Describing Rotation in Cartesian Coordinates
    - II.7.1. Abstract Motions on a Circle
    - II.7.2. Circle Actions and the Method of Coordinates on a Circle
    - II.7.3. Rotating Points about an Arbitrary Point
  - II.8. Polar Coordinates and Rotation
    - II.8.1. Fractions of a Circle and Measurement of Angles
    - II.8.2. The Sine, Cosine, and Tangent Functions
    - II.8.3. Angle Addition Formulae for Trigonometric Functions
    - II.8.4. Parameterizing Rotational Motion
    - II.8.5. Basic Surveying Problems
  - II.9. Involution
    - II.9.1. Reflections and Rotation by Half of a Circle
    - II.9.2. Inverting the Axes
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• III. Rigidity

- III.1. Lines and Planes
    - III.1.1. Introductory Comments on Rigidity
    - III.1.2. Vectors in three Spatial Dimensions
    - III.1.3. Rigidity and the Determination of Lines and Planes
    - III.1.4. Intersections of Lines and Planes
  - III.2. Polynomial Functions
    - III.2.1. Quadratic Functions and Optimization
    - III.2.2. The Factor Theorem
    - III.2.3. Sketching Polynomials
  - III.3. Rational Functions
    - III.3.1. Sketching Reciprocals of Polynomials
    - III.3.2. Asymptotic Behavior
    - III.3.3. Sketching Rational Functions
  - III.4. Solving Piecewise Rational Inequalities
    - III.4.1. Polynomial Inequalities
    - III.4.2. Inequalities Involving Rational Functions
    - III.4.3. Inequalities Involving Piecewise Rational Functions
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• IV. Symmetry

- IV.1. Introduction to Symmetry
    - IV.1.1. Invariance of Sets under a Symmetry Group
    - IV.1.2. Functions with Involutive Symmetry
  - IV.2. Translational Symmetry
    - IV.2.1. Periodicity
    - IV.2.2. Sketching Trigonometric Functions
    - IV.2.3. Inverse Trigonometric Functions
    - IV.2.4. Equations Involving Trigonometric Functions
    - IV.2.5. The Superposition of Waves
  - IV.3. Symmetric Change
    - IV.3.1. Exponential Functions and Logarithms
    - IV.3.2. Models of Symmetric Change
    - IV.3.3. The Natural Exponential and Logarithm
    - IV.3.4. Exponential Growth and Decay
  - IV.4. Scaling of Intersections
    - IV.4.1. Tangential Intersections
    - IV.4.2. Decomposition and Calculation
    - IV.4.3. Tangency and Rational Functions
  - IV.5. Reflection and Rigidity of Tangential Intersections
    - IV.5.1. An Algebraic Inverse Function Theorem
    - IV.5.2. Tangency and Extremal Values
    - IV.5.3. High Degree Intersections
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# Principles of Calculus II

## • V. Finite Approximation

- V.1. The Elementary Notion of Area
  - V.1.1. Intuition about Motion and Area
  - V.1.2. Area of Rectangles
  - V.1.3. Triangles and their Circumcircles
- V.2. Area of Polygons
  - V.2.1. Area and Orientation of Triangles
  - V.2.2. Polygonal Curves and Triangulation
  - V.2.3. The Area of a Polygon
- V.3. Sequences
  - V.3.1. Analytical Properties of the Real Numbers
  - V.3.2. Sequential Limits and the Limit Laws
- V.4. Measurement of a Circle
  - V.4.1. Fractions of a Circle
  - V.4.2. Length and Area
- V.5. Continuous Limits
  - V.5.1. Definition and Computation of Continuous Limits
  - V.5.2. One Sided Limits
  - V.5.3. Infinite Limits
  - V.5.4. Limits and Curves
- V.6. Continuous Functions
  - V.6.1. Continuity
  - V.6.2. Properties of Continuous Functions
  - V.6.3. Approximating Continuous Functions
- V.7. Analysis of Error
  - V.7.1. Asymptotic Notation
  - V.7.2. Sensitivity to Perturbation
  - V.7.3. Composite Errors
- V.8. Approximating Change
  - V.8.1. Average Rate of Change
  - V.8.2. Instantaneous Rate of Change
- V.9. Summation

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V.9.1. Infinite Series and their Convergence

V.9.2. Some Convergence Tests

V.9.3. The Exponential Function

○ V.10. Approximating Area in the Plane

V.10.1. Rectifiable Curves

V.10.2. Areas Bounded by Closed Curves

V.10.3. Approximating Area under a Function

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• VI. Local Linear Approximation of Functions

- VI.1. Approximation by the Tangent Line
    - VI.1.1. Tangency to Transcendental Functions
    - VI.1.2. Basic Differentiation Rules
    - VI.1.3. Differentiation and Decomposition
    - VI.1.4. Newton’s Method
  - VI.2. Derivatives of Elementary Functions
    - VI.2.1. Derivatives of Inverse Functions
    - VI.2.2. Implicitly Defined Functions and Their Derivatives
    - VI.2.3. Related Rates Problems
  - VI.3. Rigidity and the Local Linear Approximation
    - VI.3.1. Extreme Values and Optimization
    - VI.3.2. Mean Value Theorem
    - VI.3.3. Antiderivatives
    - VI.3.4. L’Hopital’s Rule
  - VI.4. Shape and Change
    - VI.4.1. Sketching Curves with First Order Information
    - VI.4.2. The Second Derivative
    - VI.4.3. Concavity and Curve Sketching
  - VI.5. Applications of the Mean Value Theorem
    - VI.5.1. First Order Differential Equations and Flows
    - VI.5.2. Solving Simple Differential Equations
    - VI.5.3. Uniqueness of Solutions to Certain Differential Equations
  - VI.6. Curves and Surfaces
    - VI.6.1. Particle Motion
    - VI.6.2. Curves on Simple Surfaces
    - VI.6.3. The Implicit Function Theorem
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# Principles of Calculus III

- VII. Local Higher Order Approximation
    - VII.1. Series
      - VII.1.1. Series of Functions and their Convergence
      - VII.1.2. Polynomial Approximation of Continuous Functions
      - VII.1.3. Power Series and the Radius of Convergence
    - VII.2. Higher Order Approximation
      - VII.2.1. Taylor Polynomials and Taylor’s Theorem
      - VII.2.2. Taylor Series
      - VII.2.3. Rigidity of Analytic Functions
    - VII.3. Differentiating Series
      - VII.3.1. Term-by-term Differentiation
      - VII.3.2. Application of Series
    - VII.4. The Geometry of Particle Motion
      - VII.4.1. Acceleration and Force
      - VII.4.2. Parameterizing Curves and Surfaces
      - VII.4.3. Constrained Motion
      - VII.4.4. Normal Forces
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• VIII. Integration

- VIII.1. Integration
    - VIII.1.1. The Fundamental Theorem of Calculus
    - VIII.1.2. The Integral Mean Value Theorem
    - VIII.1.3. Approximation Methods
    - VIII.1.4. Improper Integration
  - VIII.2. Beyond the Elementary Functions
    - VIII.2.1. Arc Length
    - VIII.2.2. Using the Integral to Define Functions
    - VIII.2.3. Approximation and Application
  - VIII.3. Techniques for Evaluating Antiderivatives
    - VIII.3.1. Integration by Parts
    - VIII.3.2. Bijections between Domains of Integration
    - VIII.3.3. Integration by Substitution
  - VIII.4. Integrals Involving Rational Functions
    - VIII.4.1. Reciprocals of Real Irreducible Polynomials
    - VIII.4.2. Partial Fraction Decomposition
    - VIII.4.3. Antiderivatives of Rational Functions
  - VIII.5. Trigonometric and Hyperbolic Integrals
    - VIII.5.1. Trigonometric and Hyperbolic Functions
    - VIII.5.2. Trigonometric and Hyperbolic Integrals
    - VIII.5.3. Weierstrass substitution: A framework for trigonometric substitution
    - VIII.5.4. Trigonometric and Hyperbolic Substitutions
  - VIII.6. Applications of Integration
    - VIII.6.1. Applications of Hyperbolic Functions
    - VIII.6.2. Application of Symmetry Principle: Area and Volume Integrals
    - VIII.6.3. Application of Symmetry Principle: Surface Area
    - VIII.6.4. Work Integrals
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