The Principles of Calculus I

Sample Final 1

Notes, books, calculators, and computing resources may be used in this examination, but you may not seek unauthorized assistance.

You may not receive full credit for a correct answer if insufficient work is shown.

Do not simplify your answers unless specifically requested or further than specifically requested.

The exam contains 18 questions with 156 possible points.

(108 points – C - Level)	Questions C1–C12 are each worth **nine points**.
(24 points – B - Level)	Questions B1–B3 are each worth **eight points**.
(24 points – A - Level)	Questions A1–A3 are each worth **eight points**.

C Level Questions

Problem C1.

The line L passes through the points $(2,3)$ and $(5,9)$.

(a) Find an equation for L.

(b) If $L_\perp$ is perpendicular to L, then what is the slope of $L_\perp$?

Problem C2.

Recall that 3 feet is one yard and 60 seconds is one minute. Calculate the acceleration $7\,\frac{\mathrm{ft}}{\mathrm{s}^2}$ in units of yards and minutes.

Problem C3.

Suppose that a line segment L passes through the points $(1,3)$ and $(2,5)$.

(a) Find the point on L that is a distance of 2 from the point $(1,3)$ and that lies to the right of $(1,3)$.

(b) Find the point on the line segment L whose distance from $(1,3)$ is one third the length of the line segment L.

Problem C4.

Suppose that L is a line of slope 5 that intersects the origin. Where does L intersect the unit circle?

Problem C5.

Take f to be the function given by

$$f(x) = |x+1|.$$

Write the function f explicitly as a piecewise defined function.

Problem C6.

Rotate the point $(2,3)$ by the angle $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ around the point $(1,5)$.

Problem C7.

Take f to be the function given by

$$f(x) = x^2 - 4x + 7.$$

What is the minimum y value for a point in f?

Problem C8.

Take f to be the function given by

$$f(x) = (x+2)^2(x-1)(x-4)^3.$$

Sketch f and then find all x with $f(x) \geq 0$.

Problem C9.

What is the leading term of the polynomial f given by

$$f(x) = (-2x+1)^3(3x+1)^2(x-8)^4(x-1)^{100}?$$

For some constant C and some natural number n, your answer should look like Cx^n.

Problem C10.

What fraction of a circle is an arc of $\frac{5}{3}$ radians? A Zuma-radian is a measure of an angle. There are 120 Zuma-radians in a circle. How many Zuma-radians is $\frac{5}{3}$ radians?

Problem C11.

Suppose that A and B are angles in the first quadrant and $A > B$. Put the correct symbol, $<$ or $>$, in the boxes below.

$$\cos(A) \; \square \; \cos(B)$$

$$\sin(A) \; \square \; \sin(B)$$

$$\tan(A) \; \square \; \tan(B)$$

Problem C12.

Suppose that $\log_2(A) = 3$, $\log_2(B) = 2$, and $\log_2(C) = 4$. Calculate $\log_2\left(\frac{AC^2}{B^5}\right)$.

B Level Questions

Problem B1.

Compute $(f+g)(x)$, where f and g are given by

$$f(x) = \begin{cases} 2x+5 & \text{if } x \leq -1 \\ 3x & \text{if } x > -1 \end{cases} \quad \text{and} \quad g(x) = |x+3|.$$

Problem B2.

There is a building in front of you. The angle of elevation from your position to the top of the building is $10°$. You walk 50 feet towards the building and measure the angle of elevation to now be $25°$. How tall is the building?

Problem B3.

(a) It takes 2 workers 5 hours to shovel 100 cubic feet of sand. How many hours does it take 7 workers to shovel 300 cubic feet of sand?

(b) A pyramid has a height of 5 feet and a surface area of 50 square feet. A larger pyramid with the same proportions has a height of 12 feet. What is its surface area?

(c) A pyramid has a height of 6 feet and a volume of 10 cubic feet. If a smaller pyramid of the same proportions has a volume of 2 cubic feet, what is its height?

A Level Questions

Problem A1.

At time zero, Boat A is initially at $(0, 0)$ and Boat B is initially at $(1, 4)$. Boat A has velocity vector $\langle 1, 3\rangle$ and Boat B has velocity vector $\langle -1, 2\rangle$. When are the boats closest to eachother and how far apart are they at this time? Be sure to include the proper units in your answer.

Problem A2.

Let L_1 be the line given by $y = 2x$ and let L_2 be the line given by $y = 3x + 1$. Find an equation of the line given by the reflection of L_2 across L_1. To clarify: L_1 is the "mirror" and L_2 is being reflected.

Problem A3.

Find all solutions to the inequality $|x^2 - 4| > 2$.

Principles of Calculus I
Sample Final 2

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C Level Questions

Problem C1.

Suppose that L_1 is the line given by

$$y = -x + 7$$

and L_2 is the line given by

$$y = 4x - 3.$$

Where do L_1 and L_2 intersect?

Problem C2.

Sketch on a real number line the set $(-7, 9] \cap \Big([-10, 2] \cup (5, 10)\Big)$.

Problem C3.

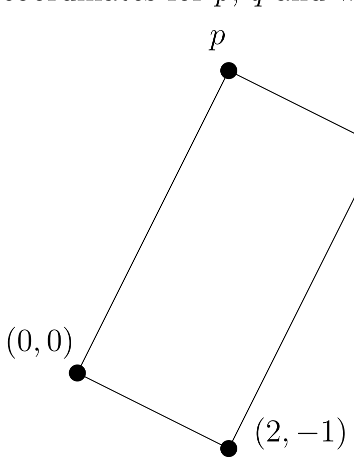
Let L be the line segment with endpoints $(1, 5)$ and $(6, 15)$. The point p lies on L. The distance from $(1, 5)$ to p is one fifth the distance of p to $(6, 15)$. What are the coordinates of p?

Problem C4.

The vector $\langle 2, 3\rangle$ translates points along the line L. The point $(1, 4)$ lies on L. Find the coordinates of the point on L that lies to the right of $(1, 4)$ and that is a distance of 7 from $(1, 4)$.

Problem C5.

The shape below is a rectangle, whose short side is half the length of the long side. Calculate the coordinates for p, q and write an equation for the line passing through p and q.



Problem C6.

Let f be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } -2 < x \leq 1 \\ 3x - 2 & \text{if } x > 3. \end{cases}$$

What is the domain of f and the range of f?

Problem C7.

Suppose that f and g are given by

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 0 \\ 3x^2 & \text{if } 0 \leq x \leq 5 \\ 6x & \text{if } x > 5 \end{cases} \qquad \text{and} \qquad g(x) = \begin{cases} \cos(x) & \text{if } x < 2 \\ x^4 + 1 & \text{if } x \geq 2. \end{cases}$$

Write $f + g$ as a piecewise defined function.

Problem C8.

Find all solutions to the equation

$$(2\sin(3\theta) - 1)(4\cos(5\theta) + 1)(\sin(\theta) + 5) = 0.$$

Problem C9.

(a) Suppose that that $A(0) = 7$ and $A(1) = 3$. If A changes exponentially with respect to time, find a formula for $A(t)$ at any time t.

(b) Suppose that that $A(2) = 7$ and $A(5) = 3$. If A changes exponentially with respect to time, find a formula for $A(t)$ at any time t.

Problem C10.

Take f to be the polynomial given by

$$f(x) = (x + 5)^7(x + 2)(x - 3)^4.$$

Sketch f and find all x with $f(x) \leq 0$.

Problem C11.

(a) Where does the line $y = 10x$ intersect the unit circle?

(b) $\cos(\tan^{-1}(10)) =$

(c) $\sin(\tan^{-1}(10)) =$

Problem C12.

Calculate:

(a) $\arccos\left(\sin\left(-\frac{\pi}{7}\right)\right)$;

(b) $\arcsin\left(\sin\left(\frac{4\pi}{9}\right)\right)$.

B Level Questions

Problem B1.

Write the feasible set of the system

$$\begin{cases} x + y \leq 1 \\ y - 2x < 4 \\ y - x \geq 1 \end{cases}$$

in set builder notation.

Problem B2.

(a) Take f to be the rational function given by

$$f(x) = \frac{(x+5)^9(x+3)^4(x-2)^3}{(x+7)^3(x-1)^2(x-6)^4}.$$

Sketch f. Be sure to emphasize the global and local behavior of f and to NOT sketch f to scale.

(b) Let h be the function given by $h(x) = |f(x)|$. Write h as a piecewise defined function.

Problem B3.

A function A is changing exponentially with respect to time. It models the decay of a certain radioactive substance. If $A(0) = 12$ and $A(5) = 7$ then what is the half-life of the substance? Units of time are measured here in hours.

A Level Questions

Problem A1.

Let f be the function defined by the formula

$$f(x) = x^3 - 4x + 1.$$

What is an equation of the line tangent to the graph of f at the point $(2, 1)$? Be sure to directly use the algebraic definition of tangency—you will earn **no credit** for using other techniques.

Problem A2.

Suppose that f and g are the functions given by

$$f(x) = \begin{cases} 5x^2 & \text{if } x < 1 \\ 2x & \text{if } x > 4 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 5 - 2x & \text{if } x < 3 \\ x - 1 & \text{if } x \geq 6. \end{cases}$$

Write $f \circ g$ as a piecewise defined function.

Problem A3.

You swim to a buoy and then back. You can swim at a speed of 2 miles per hour with respect to still water. There is a current moving from the direction of the buoy to the shore at a speed s. When you are swimming against the current, you will naturally swim more slowly with respect to the stationary buoy, but when you swim back to shore, you will move more quickly with respect to the buoy. The buoy is a distance of 2 miles from the shore. For what value of s will your total swimming time be minimal? Does your answer depend on the buoy's distance or on your speed?