Our Approach

The Curriculum

The first principle of human learning as enunciated by the National Research Council (2000) is the principle: "1. Learning with understanding is facilitated when knowledge is related to and structured around major concepts and principles of a discipline." Using guiding principles, we reduce a myriad of formulas to a handful of ideas. Our classes should help students learn principles that are as important to a freshmen student as a leading researcher and should be as interesting and informative to the students taking the class as the instructors who are teaching it. This point of view is especially significant given that in a national study, Bressoud (2014) found that only 50-60% of students successful in Pre-calculus classes continue to Calculus. See also (Sonnert and Sadler, 2014) and (Sonnert, 2015).

To address such issues, we present an innovative year-long program "*Principles of Calculus*". This program entirely reimagines the role of calculus in STEM education and is designed for a typical two-semesters calculus course in California Community College or a typical three-quarters course of pre-calculus and calculus in the California higher educational institutions. Instead of presenting mathematics in a series of facts and procedures, we organize the mathematics according to the principles of (1) decomposition, (2) transformation, (3) rigidity, (4) symmetry, (5) finite approximation, (6) local approximation, (7) higher order approximation, and (8) integration. The mathematics will be repeated applications of such principles in various settings and complexity.

Because the principles interact, the sequence is necessarily nonlinear and nonlocal in design. For example, tangency is discussed from an algebraic perspective before limits are introduced, and planar area is introduced from the perspective of finite approximation before derivatives are discussed. Because students begin studying derivatives after they already have a deep knowledge of linear motion in several variables, transformations, and vectors, they are well equipped to study velocities and accelerations associated with particles moving in two and three dimensions. A much richer variety of examples and applications in science and engineering becomes accessible to students.

Another innovation in our program is to develop students' spatial reasoning skills. Spatial reasoning skills are a set of cognitive skills that enable us to mentally manipulate, organize, reason about, and make sense of spatial relationships in real and imagined spaces (Newcombe & Shipley, 2015; Uttal et al., 2013). They are commonly used by many students already when completing everyday tasks such as assembling furniture or navigating from one location to another. Studies examining the relationship between spatial skills and mathematics achievement find that the two are significantly correlated for students at all educational levels (Casey et al., 1995; Delgado & Prieto, 2004; Geer et al., 2019). For example, mental rotation performance relates to mathematical reasoning skills in middle school students (Delgado & Prieto, 2004; Lombardi et al., 2019) and mathematical aptitude in undergraduates (Casey et al., 1995).

In addition to the performance-based evidence of the beneficial relation between spatial reasoning skills and mathematical achievement, prior research suggests that a positive relation between *spatial* and *mathematical thinking* may be based on shared cognitive processes (Gunderson et al., 2012; Mix & Cheng, 2012). Much research indicates that spatial reasoning skills are malleable and can be improved with training and practice (Uttal et al., 2013). Moreover, the development of students' spatial reasoning skills is not reliant on formal learning experiences. Researchers have found that activities such as playing blocks, puzzles, and video games facilitates the development of spatial reasoning skills (Casey et al., 2008; Jirout & Newcombe, 2015). However, when it comes to formal learning experiences, spatial reasoning has been purposefully embedded into kindergarten through undergraduate STEM curricula in only a handful of instances (Lowrie et al., 2019; Sorby, 2007). Thus, many undergraduates enrolling in gateway mathematics courses are possibly unfamiliar with how to use their spatial reasoning skills that have been developed through informal experiences to reason about complex spatial problems required to learn the subject. Understanding how to help students leverage their spatial reasoning skills within the context of mathematics learning across all aspects of a curriculum from lecture activities to video scenarios role modeling problem solving, and through collaborative discussions, we believe will generate improvement in student outcomes, especially in URM students.

- For the program outline, visit URL.

Course Materials

The course materials are designed with science relevance and cultural relevance to students in mind (Carrino & Gerace, 2016; Eaton & Highlander, 2017; Treisman, 1992). They are intended to enable active learning (Allen & Tanner, 2005; Beir et al. 2019; Eisen, 1998; Freeman et al. 2014; Goodwin et al., 1991; Graham et al., 2013; Haak et al., 2011). They are organized by adaptive technology as an e-Book and the program will be available as *Open Educational Resource*.

The learning materials are designed for four different kinds of activities: lecture, guided collaborative learning, guided discussion, and self-paced studies. Design of assessment materials are driven by formative learning process. The e-Book will contain interactive text blocks followed by activities. Activities include embedded self-check questions and exercises employing virtual manipulatives. It will contain resources in the form of embedded videos and worked out problem sets. The modular nature of the book will allow instructors to adapt course content to their instructional needs. Visualizations and animations using Desmos and Geogebra will be embedded and will allow students to interact with graphs and animations via sliders as well as build novel features into the preexisting graphical content.

- Sample lecture notes are forthcoming
- Sample animations are forthcoming

Student learnings are enforced in guided small group collaboration. The activities are driven by a weekly worksheet. Problem sets will be selected based on the concept of the Treisman Model, in which “problem sets drive the group interaction” (Treisman, 1992, p.368).

- For sample worksheet, visit URL

Using videos as micro-tutorials will introduce real world and culturally relevant examples that are being solved by student role models who are representative of the demographics of the students in the class. The goal is to allow self-paced learning (Rahman et al., 2015) which provides scaffolded exercises that are designed to raise student self-efficacy.

- For sample videos, visit https://microtutorials.ucr.edu .

Assessments include midterm and final exams, activities, peer evaluation of problem sets, computer driven problem sets using reality-based scenarios, and class participation which will be determined by student work on low stakes daily quizzes. All assessments excluding the midterm and final exams are formative. As all formative assessments are enabled by adaptive learning technology we could afford students' choice for discipline-based applications, self-checking quiz, and peer review.

- For sample assessment, visit URL.

Adaptive Technology

This project's adaptive technology embedded within the curriculum builds on the work of Mayer (2005) which indicates that adaptive strategies that include imagery and visualization (such as video tutorials or computer simulations) have been proven to allow students to learn and recall information better than from words alone (Pilegard & Mayer, 2016). This is especially true for content that is abstract, such as in a calculus course whose principles can be hard to visualize without grounding in real life imagery. For example, Stieff (2011) found that computer-based visualization that generated simulations supported student learning and representational competence in STEM topics (Stieff, 2011; Tasker & Dalton, 2006). Research further suggests that adaptive strategies can play a significant role in student mathematical word-problem-solving accuracy especially for those whose memories are taxed by low memory capacity (Swanson, et al. 2013; Doolittle et al., 2009; Service, et al, 2002) when problems are especially cognitively demanding (Park et al, 2011).

To address such needs, we produce an open online interactive eBook built as an IMS Common Cartridge file that plugs into Canvas and can be shared in the Canvas Commons. The book will contain interactive text blocks followed by activities. Activities include embedded self-check questions and exercises employing virtual manipulatives. It will contain resources in the form of embedded videos and worked out problem sets. It will leverage the Outcomes functionality in Canvas by tagging each instructional element with its corresponding learning outcome to track student performance in alignment with course. The modular nature of the book will allow instructors to adapt course content to their instructional needs.